Max. non-expansion work

$H = U + pV$, so $dH = dU + d(pV) = dq + dw + d(pV)$

$G = H - TS$, so $dG = dH - TdS - SdT$

$$= dq + dw + d(pV) - TdS - SdT$$

$\Delta T = 0$, isothermal:

$$dG = dq + dw + d(pV) - TdS$$

Reversible: $dw = dw_{rev}$, $dq = dq_{rev} = TdS$

so, $dG = dw_{rev} + d(pV)$

→ expansion work: $w = -pdV$

→ non-expansion work: w_{ne} (e.g., electrons in a wire, raising a column of liquid vs. gravity, batteries (electrochem.), etc.)

$\therefore dG = dw_{ne} - pdV + pdV + Vdp$

$dG = dw_{ne} + Vdp$

if change happens at const. p, T (necessary for G), then

$$dG = dw_{ne}$$

For a reversible process, work done is the max value, so

$dG = dw_{ne,max}$, $\Delta G = w_{ne,max}$

qed.