

Problem 1.23 (7th); 1.20 (8th)

$$V_m = f(T, p), \text{ so } dV_m = \left(\frac{\partial V_m}{\partial T}\right)_p dT + \left(\frac{\partial V_m}{\partial p}\right)_T dp$$

$$\left(\frac{\partial V_m}{\partial T}\right)_p \left(\frac{\partial T}{\partial p}\right)_{V_m} \left(\frac{\partial p}{\partial V_m}\right)_T = -1, \text{ so } \left(\frac{\partial V_m}{\partial T}\right)_p = - \frac{\left(\frac{\partial p}{\partial T}\right)_{V_m}}{\left(\frac{\partial p}{\partial V_m}\right)_T} \leftarrow \text{inversion rule}$$

CHAIN RULE \uparrow

$$\left(\frac{\partial p}{\partial V_m}\right)_T = \frac{-RT}{V_m^2} - \frac{2(a+bT)}{V_m^3}$$

$$\left(\frac{\partial p}{\partial T}\right)_{V_m} = \frac{R}{V_m} + \frac{b}{V_m^2}$$

$$\left(\frac{\partial V_m}{\partial T}\right)_p = \frac{R + \frac{b}{V_m}}{\frac{RT}{V_m} + \frac{2(a+bT)}{V_m^2}}$$

from the equation of state:

$$\frac{a+bT}{V_m^2} = p - \frac{RT}{V_m}$$

$$\text{so, } \left(\frac{\partial V_m}{\partial T}\right)_p = \frac{\left(R + \frac{b}{V_m}\right)}{\frac{RT}{V_m} + 2\left(p - \frac{RT}{V_m}\right)} = \frac{R + \frac{b}{V_m}}{2p - \frac{RT}{V_m}}$$
$$= \boxed{\frac{RV_m + b}{2pV_m - RT}}$$