

59-240  
Lecture 4  
Real Gases

Principle of Corresponding States

Flat inflection: 2 derivatives equal to zero

critical constants are expressed in terms of vdW coefficients

reduced variables used to show all gases behave the same

Molecular Interactions

Low pressure: like a perfect gas

Intermediate pressure: Attractive forces

High pressure: Repulsive forces

$$Z = \frac{pV_m}{RT}$$

Compression factor

Ideal gas:  $Z=1$

Interm. pressure:  $Z < 1$

High pressure:  $Z > 1$

Different gases have different responses

van der Waals

analytical equation

semiempirical theory

experimental observations

$nb$ : small approximate volume occupied by the molecules

$a$ : positive proportionality constant (attractive/repulsive forces)

vdW loops/Maxwell construction

$$p = \frac{nRT}{V - nb} - a\left(\frac{n}{V}\right)^2 = \frac{RT}{V_m - b} - \frac{a}{V_m^2}$$

Virial Equations

Power series, empirically measured

Numerical equation

$$pV_m = RT(1 + B'p + C'p^2 + \dots) = RT\left(1 + \frac{B}{V_m} + \frac{C}{V_m^2} + \dots\right)$$

Perfect gas:  $p \rightarrow 0$

$p$  increases:  $B$  kicks in, linear relation btw  $Z$  and  $p$

$p$  high:  $C$  and higher order terms, not linear

Virial equations are **force** equations

Condensation of CO<sub>2</sub>

20 C example

Vapour liquid line

31.04 C example

Critical temperature

Critical constants,  $p_c$ ,  $V_c$ ,  $T_c$

Supercritical temperature,  $T_c$

Boyle Temperature

Temperature at which  $dZ/dp = 0$  as  $p \rightarrow 0$

$$\frac{dZ}{dp} = B' + 2pC' + \dots \rightarrow B' \text{ as } p \rightarrow 0$$

Low  $T$ :  $dZ/dp < 0$ ,  $B$  -ve

High  $T$ :  $dZ/dp > 0$ ,  $B$  +ve

Boyle  $T$ :  $B = 0$

Virial Coefficients

$$pV_m = RT(1 + B'p + C'p^2 + D'p^3)$$

Methane example