Example 1B.1 Calculating the mean speed of molecules in a gas

Calculate the mean speed, v_{mean}, of N_2 molecules in air at 25 °C.

Method The mean speed is obtained by evaluating the integral

$$v_{mean} = \int_0^\infty v f(v) dv$$

with $f(v)$ given in eqn 1B.4. Either use mathematical software or use the standard integrals in the *Resource section*.

Answer The integral required is

$$v_{mean} = 4\pi \left(\frac{M}{2\pi RT} \right)^{3/2} \int_0^\infty v^3 e^{-mv^2/2kT} dv$$

$$\overset{\text{Integral G.4}}{=} 4\pi \left(\frac{M}{2\pi RT} \right)^{3/2} \times \tfrac{1}{2} \left(\frac{2RT}{M} \right)^{1/2} = \left(\frac{8RT}{\pi M} \right)^{1/2}$$

This should be 2 (i.e., squared)

Substitution of the data then gives

$$v_{mean} = \left(\frac{8 \times (8.3145 \text{ J K}^{-1} \text{ mol}^{-1}) \times (298 \text{ K})}{\pi \times (28.02 \times 10^{-3} \text{ kg mol}^{-1})} \right)^{1/2} = 475 \text{ m s}^{-1}$$

We have used 1 J = 1 kg m^2 s^{-2} (the difference from the earlier value of 474 is due to rounding effects in that calculation; this value is more accurate).

Self-test 1B.2 Evaluate the root-mean-square speed of the molecules by integration. Use mathematical software or use a standard integral in the *Resource section*.

Answer: $v_{rms} = (3RT/M)^{1/2} = 515 \text{ m s}^{-1}$